Network models of non-equilibrium growth

Abstract

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The structures of Nature are often shaped by growth processes. Some owe their form to the inhomogeneity of the constitutive matter. Frequently however the unique pattern of a final structure is determined by fluctuations in the medium, amplified by the growth dynamics. Structures of that kind occur in numerous processes, such as: electrochemical deposition, slow combustion, crystallisation, or the growth of fibres and microtubules. The dissertation 'Network models of non-equilibrium growth' is devoted to two particular cases: pattern formation in a dissolving porous medium, and the viscous fingering process, where a high-viscosity fluid is displaced by a less viscous one.

The growth process in a wide variety of systems may be described in terms of a harmonic scalar field $p(x, t)$, interpreted for instance as a reagent concentration or pressure. Despite the linearity of the Laplace equation, the growth problem becomes non-linear due to the boundary conditions – the front is unstable under small perturbations. Hence, even though the basic mechanisms of growth are well understood, the strongly non-linear character of the processes at the phase boundary makes an analytic description far from being simple. A good description exists for the early stages of growth, where linear stability analysis allows one to find the perturbation modes undergoing strongest amplification.

The main focus of the present work are the much less understood later stages of growth, where the system is occupied by multiple finger-like structures evolved from the initial instabilities. These structures interact with each other, compete for growth, and branch into more and more complex forms. In the viscous fingering case, a rectangular lattice of channels hosting a pair of fluids is used to obtain a system featuring numerous relatively thin fingers. In the porous medium dissolution case, the width and type of emerging pattern depends directly on the so-called Damköhler number – a function of the flow, reaction rate and pore size. In effect one may choose the parameters of the system so as to produce numerous thin, fingerlike structures (so called 'wormholes'), with or without branching.

Systems exhibiting thin finger structures have been investigated in a series of microfluidic experiments in collaboration with the Institute of Physical Chemistry PAS. Subsequently, a nu-

merical model has been developed. For the purposes of the latter model, the network of channels (pores) is viewed as a network of resistors whose resistances evolve in time. In the viscous fingering case, the resistance depends on the viscosity of the liquid occupying a given channel; in the porous medium dissolution case, it is a decreasing function of the pore diameter (the diameters grow with time due to dissolution). Furthermore, in the latter case we allow several channels to merge into one, leading to a dynamical evolution of the network topology.

Fingers of various widths have been achieved both experimentally and numerically, including extremely thin ones, whose width is of the same order as the channel spacing. The dependence of the finger/wormhole shape on the parameters of the system has been found for both viscous fingering and porous medium dissolution. In presence of several fingers, their interactions play a significant role. Most importantly, they lead the fingers to compete for the available flow, with longer fingers screening their shorter neighbours. This results in a self-similar pattern of fingers whose lengths obey a power-law distribution. The analysis of finger/wormhole lengths for viscous fingering and porous medium dissolution shows that the respective distributions are comparable.

There are also attractive and repulsive interactions between the fingers, manifesting themselves in finger shape asymmetry. A universal resistor model has been developed in order to describe this phenomenon, explaining the dependence of interaction strength and direction on the length of the fingers, as well as their resistance relative to the bulk. The model yields correct results for both viscous fingering and porous medium dissolution.

In the course of our experiments involving a pair of miscible fluids flowing through a lattice of channels, we have observed a new phenomenon where the heads of the fingers become detached. The causes of this behaviour have been analysed, producing a relation between the size of the detached heads and the parameters of a system.

A separate problem in the case of wormhole formation in porous media is to find the optimal conditions for dissolution, minimising the amount of reagent necessary for a wormhole to penetrate the entire system (so-called break-through). We have investigated how the reagent volume needed for break-through varies with the parameters of the system. A non-trivial dependence has been established, with different local minima corresponding to different dissolution regimes.